Shear Induced Jamming in Granular Pastes

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Granular Systems

“The processing of granular materials consumes roughly 10% of all the energy consumed on this planet” (J. Duran)

Solid and Liquid in the same time
Rheology: what happens in the sample?
Rheology

Shear stress: \( \tau = \frac{F}{S} \)

Shear rate: \( \dot{\gamma} = \frac{V}{H} \)

Macroscopic measurement: \( F(V) \) (or \( T(\Omega) \))

Hypotheses:

- homogeneous shear
- homogeneous material

Constitutive law: \( \tau(\dot{\gamma}, x) \)

Bad evaluation of shear rate
Bad knowledge of the material

vendredi 9 décembre 2011
Viscosity measurement in granular suspensions

Measurements with 5 different rheometers

![Graph showing viscosity measurement](image)

Ferraris et al., NIST (2001)
Shear thickening happens at high shear rate

This macroscopic behavior is it the one that reflects the intrinsic rheological behavior of the material?
Dense viscous granular suspensions

Polystyrene beads: ø 40 µm, ρ = 1.05 g.cm⁻³

Water + Cesium chloride: ρ = 1.05 g.cm⁻³, η = 1 mPa.s

Volume fraction: 57 to 60%

Geometry:
Wide gap Couette (R_{inner}=4cm, R_{outer}=6cm)

Surfaces covered with sandpaper
→ avoid wall slip
Local measurements with the Magnetic Resonance Imager

Torque measurement: rheometer

Velocity profile $V(R)$ measurement

Volume fraction profile $\phi(R)$ measurement
Torque measurements

in a 59% matched suspension

Jump ↔ Shear thickening

Hysteresis ↔ Shear induced migration

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Profile independent of velocity: measurements performed on an homogeneous material on a stationary structure.
Migration generates a JAMMED ZONE = concentration near the maximum packing fraction $\phi_m \Rightarrow$ Shear localization at high shear rates.
Shear induced migration: a sketch (for rigid particles)

**Diffusion theory** (Leighton and Acrivos 1987, Phillips et al. 1992):

\[ \gamma(x + \Delta x) \approx \gamma(x) \]

\[ \text{collision rate} \propto \dot{\gamma} \]

⇒ flux induced by gradients in shear rate
development in any heterogeneous flow (Couette, Poiseuille, etc.)
Shear induced Migration in viscous suspension

Theories:

**Diffusion** (Leighton and Acrivos 1987, Phillips et al. 1992):
flux induced by gradients in shear rate

**Normal stresses** (Nott and Brady 1994, Mills and Snabre 1995):
particle momentum (=flux) counterbalances the gradients in normal stresses

\[
D = \bar{D}(\varphi) a^2 \dot{\gamma}
\]

Experimentally \(\bar{D}(\varphi)\) varies slowly with \(\varphi\) up to \(\varphi = 55\%\)

Diffusion \(\Rightarrow\) strainscale \(\propto \frac{(\text{gap})^2}{(\text{particle size})^2}\)

**Strainscale** independent of shear rate and large: strainscale expected \(\sim 10000\)
Near the packing: Puzzling observations

59% suspension of 40 microns beads

Strain at the end of migration

- Slow at low shear rate (strain > 1000)
- “Instantaneous” at high shear rate (strain < 50)

Macrophscopic shear rate (s⁻¹)

Rotational speed Ω (rpm)
Shear induced migration in viscous suspension

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Shear induced migration in viscous suspension

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\[ D = \bar{D}(\phi) a^2 \dot{\gamma} \]

Experimentally \( \bar{D}(\phi) \) varies slowly with \( \phi \) up to \( \phi \approx 35\% \)

**Diffusion \Rightarrow strainscale \approx \frac{(\text{gap})^2}{(\text{particle size})^2}**

Strainscale independent of shear rate and large: strainscale expected \( \sim 1000 \)

Near the maximum packing: “instantaneous” migration (less than 100 revolutions)
Local behavior: Transient regime

\[ \dot{\gamma}(r) = \left. \frac{\partial v}{\partial r} \right|_{r} - \frac{v}{r} \]

\[ \tau(r) = \frac{T}{2\pi H r^2} \]

\[ \Rightarrow \tau = f(\dot{\gamma}) \]

The discontinuous shear thickening observed during the initial ramp-up is a transient phenomenon associated with a large scale reorganization of the material, which involves shear induced migration from low to high shear zones.
Steady-state local behavior

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\[ \dot{\gamma}(R, \Omega) = r \frac{\partial}{\partial r} \left( \frac{V}{r} \right) \]

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\[ \Rightarrow \phi = f(R, \Omega) \Rightarrow R \leftrightarrow \phi \]

Profiles independent of velocity (once heterogeneous)
\[ \Rightarrow \text{at radius } R, \text{ same material for all } \Omega \]
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Local behavior at constant volume fraction 59%

Fall et al. PRL (2010)

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Shear thickening: Viscous to Granular transition

Local measurement at constant volume fraction

- **Viscous forces + Contact forces**; negligible inertia
- **Contact forces + Grains inertia**; negligible viscous forces


\[ \tau \sim \dot{\gamma} \quad \text{and} \quad \tau \sim \dot{\gamma}^2 \]
Near the maximum packing fraction

**Viscous**

Viscous forces + Contact forces
negligible inertia

\[ 0 = \sum_j F_{ij} + F_i^{visc} \]

\[ F_{ij} \propto \dot{\gamma} \]

\[ \tau = \eta_0 \dot{\gamma} \sum_v (\varphi) \]

with \( \sum_v (\varphi) \propto (\varphi_m - \varphi)^{-1} \)

Near the jamming: \( \varphi \to \varphi_m \)

\[ \tau_c = \left( \frac{\eta_0^2}{\rho d^2} \right) \quad \dot{\gamma}_c = \frac{\eta_0}{\rho d^2} (\varphi_m - \varphi) \]

**Granular**

Contact forces + Grains inertia
negligible viscous forces

\[ m \frac{\partial^2 r_i}{\partial t^2} = \sum_j F_{ij} + F_i^{visc} \]

\[ F_{ij} \propto \dot{\gamma}^2 \]

\[ \tau = \rho d^2 \dot{\gamma}^2 \sum_l (\varphi) \]

with \( \sum_l (\varphi) \propto (\varphi_m - \varphi)^{-2} \)

\[ \text{Near the jamming: } \varphi \to \varphi_m \]

Peyneau (2009)

Mills & Snabre (2009)


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Difference in singular behavior of the inertial and viscous stresses at the approach of jamming which leads to the **linear vanishing of the critical shear rate** and hence permits this transition to take place at **low strain rates**.
...and the normal stresses?

Quadratic evolution $N \sim \dot{\gamma}^2 \Rightarrow$ Frictional behavior since the shear stress has the same scaling
Shear thickening conclusion

Shear thickening is due to the emergence of a granular rheology

Its main effect is to lead to Reynolds dilatancy, entailing:

- Normal forces
- Confinement effects
- Extremely fast particle migration

Which then lead to shear thickening.......
Thanks

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WZI-University of Amsterdam

François Bertrand, Anaël Lemaître and Guillaume Ovarlez

Université Paris-Est, Laboratoire Navier

vendredi 9 décembre 2011